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The theory on 'gravity-driven horizontal locomotion' is flawed; a commentary on 'Gravity-driven horizontal locomotion: theory and experiment' by Kanstad & Kononoff (2015)

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In a recent paper, Kanstad & Kononoff (*Proc. R. Soc. A* **471**, 20150287. (doi:10.1098/rspa.2015.0287)) presented a theoretical analysis of the mechanical energetics of a particular style of human walking and running. According to their analysis, the force of gravity provides energy when this style of horizontal walking/running is adopted. Furthermore, Kanstad & Kononoff suggested that uphill walking at zero energy cost is possible when the suggested style of walking is adopted. In this commentary, we argue that these claims violate the basic laws of thermodynamics, and are based on erroneous application of the basic laws of classical mechanics.

1. Commentary

In their recently published paper 'Gravity-driven horizontal locomotion: theory and experiment', Kanstad & Kononoff [1] (referred to as KK2015 for brevity) provide a theoretical analysis regarding the mechanical energetics of a particular style of human walking, and present preliminary data intended to support their predictions. In short, the proposed style of

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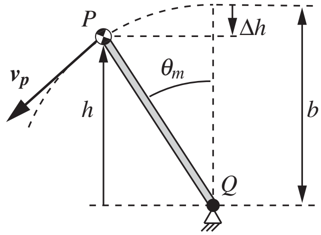

Figure 1. Schematic representation of the point mass inverted pendulum model used by Kanstad & Kononoff [1] to analyse the energetics of the stance phase of KK-walking. This figure is based on fig. 2 in Kanstad & Kononoff [1] and concerns walking/running from right to left. During a stance phase of KK-walking, the angle θ increases monotonically from zero to a final value of θ_m rad, resulting in a lowering of the point mass *P* (with mass *M* kg) by Δh m. Note that these changes do not represent the full gait cycle. In particular, at the start of the next stance phase, *h* needs to be restored to its initial value of *b* if horizontal walking is to be sustained.

walking, that we will refer to as KK-walking for brevity, entails placement of the foot of the leading leg directly below the body centre of mass at heel strike, as opposed to the commonly used placement anterior to the body centre of mass. As argued in KK2015, when modelling the human body as an inverted pendulum the proposed foot placement results in a stance phase during which the vertical coordinate of the body centre of mass decreases monotonically. As a result, the gravitational potential energy of the body decreases monotonically throughout the stance phase; the idea is illustrated in figure 1, which is similar to fig. 2 in KK2015. In this commentary, we focus on the theoretical analysis presented by KK2015. We will show that their main theoretical claims are incompatible with the laws of thermodynamics, and subsequently will show at which point their mechanical analysis is flawed.

Let us start by providing four quotes from KK2015, to establish that our concerns are not fuelled by a single, perhaps unintentional, formulation, but in fact concern the core of the theoretical claims made by the authors:

> '*Horizontal walking may become entirely driven by angular momentum generated by the field of gravity at a certain low velocity*' (1. Introduction)
>
> '*A new concept has been identified, in the shape of a velocity at which walking is entirely driven by angular momentum generated by the force of gravity*' (6. Discussion and conclusion)
>
> '*Walking at higher velocities than v_g will require muscular energy to increase the speed, while walking at lower velocities would need muscular force to retard the motion, as ΔE_{rot} would then be larger than E_v and gravity would tend to increase the speed towards v_g*' (4. Walking)
>
> '*For inclinations of $\alpha < 0.5\phi_m$, it might be feasible to walk uphill without muscular effort, being fully driven by angular momentum generated by the field of gravity*'. (6. Discussion and conclusion)

A basic grasp of classical mechanics suffices to realize that the latter claim ('uphill walking can be driven by gravity') must be incorrect. Under the best-case assumption that no energy is degraded into heat during uphill walking, the unavoidable increase in gravitational potential energy of any *passive system* must equal the corresponding decrease in total kinetic energy (see below for a precise definition) of that passive system. Thus, constant-velocity uphill walking is impossible in a *passive* system. Furthermore, in a system actuated by muscles only, constant-velocity periodic uphill walking requires that the net mechanical muscle work done over one stride is at least equal to the gravitational potential energy gained by the body during that stride, irrespective of the style of walking. Put differently, if KK-walking would indeed allow a passive system to walk steadily uphill, then KK-walking would constitute a system that outperforms a *perpetual motion machine*, and would thus violate the laws of thermodynamics.

Regarding horizontal walking and running, the claim of KK2015 is that, when adopting the proposed style of walking/running, the 'rotational energy' (see below to understand why we use quotation marks) gained by the body during the stance phase (figure 1) may be used to drive walking. In the words of the authors: *'The runner leaves the ground with rotational energy* δE_{rot} *... that is superimposed on the forward motion characterized by ... kinetic energy* $E_v = (0.5)Mv^2$ *.... Instantly upon landing, therefore, ... an amount of energy* δE_{rot} *is added to the external kinetic energy* E_v*''.*

According to KK2015, when using KK-walking, there is a walking velocity at which the 'rotational energy' liberated during each stance phase 'matches' the required kinetic energy of the body centre of mass; in the absence of mechanical energy input from muscles, the system would converge to this velocity, driven by the force of gravity (see the third quote above).

To understand that this claim must be incorrect, we first consider the total amount of mechanical energy buffered in the human body at any instant in time, neglecting energy storage in elastic structures as this is not relevant for the present discussion. As discussed for example by van Ingen Schenau & Cavanagh [2], this total mechanical energy can be decomposed into two terms (using the symbols from KK2015 wherever possible):

$$E_{\mathrm{tot}} = E_h + E_{\mathrm{kin,tot}}. \tag{1.1}$$

The first term on the right-hand side (E_h) represents gravitational potential energy; $E_{\mathrm{kin,tot}}$ represents the kinetic energy of all body parts combined. Thus, the change in the total mechanical energy buffered in a human body of total mass M that occurs during an arbitrary motion from state *0* to state *f* (again neglecting energy storage in elastic structures) can be expressed as follows (e.g. [3]):

$$\begin{aligned}\Delta E_{\mathrm{tot}} &= \Delta E_h + \Delta E_{\mathrm{kin,tot}}\\ &= -M\cdot g\cdot(h_f - h_0) + \Delta\int_0^M 0.5\cdot|\boldsymbol{v}_m|^2\,\mathrm{d}m.\end{aligned} \tag{1.2}$$

As in KK2015, h is defined as the vertical coordinate of the body centre of mass (upwards positive) and g (taken to be negative) is the gravitational acceleration. The integration over mass in the second term simply indicates that the total kinetic energy of a system equals the sum of the kinetic energies of all points of that system. The symbol $\boldsymbol{v}$ represents the velocity vector.

We now consider the convergence towards the optimal horizontal walking velocity v_g, starting from a velocity lower than v_g, which will occur according to KK2015; we focus on the energetic changes. Adopting the point mass inverted pendulum model of KK2015 (figure 1), we agree with KK2015 that, if their claim is correct, the kinetic energy of the point mass at a well-defined point in the gait cycle (e.g. heel strike) would increase over steps during this convergence. However, it is clear that during sustained horizontal walking/running, h at heel strike is essentially the same in each step (notwithstanding fluctuations in h within each step cycle). Thus, over a number of step cycles, ΔE_h (the net change in gravitational potential energy) must equal zero. Consequently, the proposed increase in the kinetic energy of the point mass at heel strike over a number of step cycles cannot be due to a decrease in gravitational potential energy between the first and last heel strike considered. Irrespective of the style of walking adopted, it cannot be true that gravity drives horizontal walking/running in the energetic sense suggested by the authors.

At what point, then, is the theoretical analysis of the authors flawed? To answer this question, let us follow the argumentation of the authors. Their derivation concerns the stance phase of KK-walking, the essential aspect of which is that at heel strike, $\theta = 0$ rad (figure 1). During the stance phase, the rigid pendulum (with length b) rotates passively from $\theta = 0$ rad (pendulum oriented vertically) to $\theta = \theta_m$ rad at toe-off. KK2015 equation (1.1) correctly captures the corresponding (negative) change in gravitational potential energy:

$$\Delta E_h = -M\cdot g\cdot\Delta h = M\cdot g\cdot b\cdot(1-\cos(\theta_m)), \tag{1.3}$$

KK2015 then continue to analyse, for this stance phase, the (linearized) change in angular momentum of the inverted pendulum relative to the point of support, labelled Q here (figure 1). From that analysis, they derive an expression (KK2015 equation 2.2) for the (linearized) change in 'rotational energy' relative to point Q, yielding:

$$\Delta E_{\text{rot}} = \Delta(0.5 \cdot I_{/Q} \cdot \omega^2) = -M \cdot g \cdot b \cdot (1 - \cos(\theta_m)). \tag{1.4}$$

In this equation, $I_{/Q}$ is the moment of inertia of the point mass pendulum relative to point Q, which is obviously equal to $M \cdot b^2$ and $\omega = \mathrm{d}\theta/\mathrm{d}t$. Apart from an error in the signs that we have corrected here (KK2015 equations (1.1) and (2.2) suggest that if ΔE_h is negative, then ΔE_{rot} is also negative, which is definitely not what KK2015 intend to claim), equation (1.4) indicates, as noted by KK2015, that the loss in gravitational potential energy during the stance phase equals the gain in 'rotational energy'. At this point, however, the authors erroneously claim that this 'ΔE_{rot} *is intrinsic to the body,* i.e. *not connected with the forward motion*'. In other words, the authors erroneously claim that this 'rotational energy', gained by the body during the stance phase, is independent of the change in kinetic energy of the point mass P. This then leads the authors to suggest that while the kinetic energy of the point mass P is continuously exchanged with gravitational potential energy, an amount of energy ΔE_{rot} is 'created' during each stance phase, that can subsequently be used to 'drive locomotion' in an energetic sense.

It is in claiming that the increase in ΔE_{rot} during the stance phase is '*not connected with the forward motion*' where KK2015 have failed to apply classical mechanics correctly. In particular, KK2015 have failed to realize that, due to the fact that their point mass pendulum has one mechanical degree of freedom (DOF), the angular velocity of the pendulum and the velocity of the point mass P are mutually dependent. As a result of this mutual dependency, $\Delta E_{\text{kin,tot}}$ (the change in total kinetic energy during the stance phase) can be expressed *either* as a function of the point mass velocity, *or* as a function of the pendulum angular velocity. The relevant expressions can be readily derived from the general expression for $\Delta E_{\text{kin,tot}}$ (equation (1.2)) and basic kinematic relations. From equation (1.2), it is immediately clear how the change in total kinetic energy of a point mass pendulum can be expressed in terms of the velocity of the point mass P (representing the only mass in the system):

$$\Delta E_{\text{kin,tot}} = \Delta E_{\text{kin},P} = \Delta \int_0^M 0.5 \cdot |\boldsymbol{v_m}|^2 \,\mathrm{d}m = \Delta(0.5 \cdot M \cdot |\boldsymbol{v_p}|^2). \tag{1.5}$$

For this 1-DOF system, the magnitude of the point mass velocity during the stance phase is related to the magnitude of the angular velocity of the pendulum:

$$|\boldsymbol{v_p}| = |b \cdot \omega|. \tag{1.6}$$

Substitution of equation (1.6) in equation (1.5) yields:

$$\Delta E_{\text{kin,tot}} = \Delta(0.5 \cdot M \cdot |\boldsymbol{v_p}|^2) = \Delta(0.5 \cdot M \cdot |b \cdot \omega|^2) = \Delta(0.5 \cdot M \cdot b^2 \cdot \omega^2) = \Delta(0.5 \cdot I_{/Q} \cdot \omega^2). \tag{1.7}$$

In sum, it follows immediately from equation (1.2), applied to the stance phase considered by KK2015, that $-\Delta E_h$ equals $\Delta E_{\text{kin,tot}}$, and it follows from equation (1.7) that $\Delta E_{\text{kin,tot}}$ may be expressed *either* as $\Delta E_{\text{kin,tot}} = \Delta(0.5 \cdot M \cdot |\boldsymbol{v_p}|^2)$, *or* as $\Delta E_{\text{kin,tot}} = \Delta(0.5 \cdot I_{/Q} \cdot \omega^2)$, being two equivalent expressions for the change in total kinetic energy.

In sum, the claim in KK2015 that the gravitational potential energy released during each stance phase of KK-walking results in a gain in 'rotational energy' that comes on top of the gain in point mass kinetic energy, and that this surplus energy can drive locomotion, is simply wrong. Instead, in a best-case scenario (when no energy is dissipated at all) the gravitational potential energy released during a stance phase is entirely buffered as kinetic energy. This kinetic energy can either be expressed in terms of point mass velocity or, equivalently, in terms of pendulum rotational energy. At the very best, this kinetic energy suffices to restore h to its original value at the start of the next step cycle; there is no surplus energy to be carried to the next step under any circumstances.

Thus, even if convincing experimental evidence for energetic advantages of KK-walking would be available, the explanation for such findings is not to be found in the theoretical analysis presented in Kanstad & Kononoff [1].

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